Inventory Models and Cost Variables

Albert T.Y. Kuo

1. *Introduction.*

It is necessary for an enterprise to have stock in order to make its business smooth and it's operation effective. If a shopkeeper has no stock he will lose his customer's goodwill by poor service, and the same for a factory; if it does not stock essential components so that the whole factory will be idled down.

An inventory can be defined as a stock of goods which is held for the purpose of future production or sales. Historically the inventory was grouped into three classes. They are: a) Raw material inventory deals with those stocks before the whole manufacturing process. b) Work-in-process inventory deals with those stocks between manufacturing processes; and c) Finished goods inventory deals with those stocks after the whole manufacturing process. The purpose of any stock is to save money and to increase the efficiency of production. It is usually necessary to hold a certain amount of inventory, but we know if there is a big change in its inventory level the efficiency of the enterprise also changed. It is very important to have proper control on the inventory, since improper control will result in loss of money and decrease efficiency. The course of action related to the inventory control is to change the period of the production run or change its size, but this will be influenced by any change in sales promotion or other marketing activities. These are looked upon as constants, or neglected as matters beyond control. It is not difficult to think about the advantages of increasing the inventory and its disadvantages. The advantages will be economy by long uninterrupted stable production runs because these runs reduce setup costs and hence minimize manufacturing costs, stabilize labor, make possible immediate delivery, increase revenue by market inflation, etc. The disadvantages will be the increase of inventory holding cost which is proportional to the size of inventory, including interest, rent, physical account expenses, etc. and obsolescence is also one of the disadvantages. Inventory policy is going to weigh and balance these two conflicts. Operations research is very interested in this inventory problem. It is called an executive-type problem, because it involves a) the effectiveness of the organization as a whole and b) a conflict of interest among the functional units of the organization. The choice among the inventory policies depends upon their "profitability" or "effectiveness" which is a function of the variables subject to control. Those cost variables that determine this effectiveness are. 1) Ordering or manufacturing cost 2) Holding or carrying cost. 3) Unsatisfied demand or shortage penalty cost. 4) Salvage cost. 5) Discount rates, etc.

Now let's make a practical determination of each of these cost variables by the use of some models of economic lot size.

2. *Optimizing Lot Size.*

The most common inventory problem faced by industry concerns the situation where stock levels are depleted with time and then are replenished by the arrival of new items. The decisions concerning inventory levels can be classified as a) The time at which orders for goods are to be placed is fixed and the determination of the quantity to be ordered, and b) both order time and order quantity should be determined. Our concern is to reach an optimum decision that is the one which minimizes the sum of the costs associated with inventory.

Much has been written about the formulae (mathematical models) for the determination of the economic lot size for purchasing and manufacturing. They differ from each other because some are more elaborate. That means, some consider a greater number of variables than others, and some use a different assumption. When we consider the economic lot size we have to determine what variables we take and what elements we neglect, if they are not the function of the lot size. It is useless if we use those variables which have a little relationship to the model, since the effect will be very small. We muct notice that we can not apply the cost accounting figure directly to the value of the model. From the accounting point of view, it may be necessary to include a share of the indirect manufacturing expense in the inventory value. However, this figure is not a function of the lot size and so may be neglected in the formulation of the model.

The simplest model representing the situation is given by F. W. Harris back to 1913 known as the Harris formula which minimizes the sum of inventory holding cost and setup costs where demand was known and constant. Actually, similar studies were made by Lord Kelvin. It was first pointed out by Lord Kelvin that the economical size of a conductor is that for which the annual investment charges just equal the annual cost of lost energy. This is well known in electrical engineering as Kelvin's Law.

In the following models we deal with discrete units only.

2. 1. *Economic Lot Size, No Shortage.*

Suppose items are withdrawn continuously and at a known constant rate, denoted by a. It is further assumed that items are manufactured in equal number and let Q represent the production run size. The production is instantaneous. It will first be assumed that shortages are not to be permitted. The only costs to be considered here are:

K=the setup cost and take down cost per production run, charged at the begining and the end of the period

c=the unit cost per item

h=the inventory holding cost per item per unit of time.

The manufacturer's inventory problem is to determine 1) how often to make a production run and, 2) of what size it should be made per run in order that the cost per unit of time is a miunimum.

The situation just described can be summarised in Fig. 1

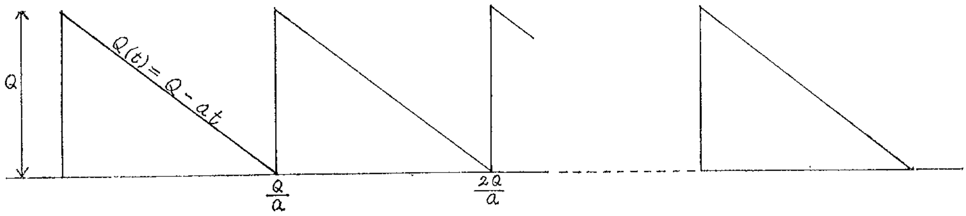

Fig. 1

The production cost per run is given by

$$\begin{cases} 0 & \text{if } Q=0 \\ K+cQ & \text{if } Q>0 \end{cases}$$

and the holding cost per run is given by

$$h\int_0^{Q/a} Q(t)dt = h\int_0^{Q/a} (Q-at)dt = h\Big[Qt - 1/2\ at^2\Big]_0^{Q/a} = h\Big[Q(Q/a) - 1/2\ a(Q^2/a^2)\Big] = hQ^2/2a$$

Therefore the total expected cost per run is

$$K+cQ+hQ^2/2a$$

And the total expected cost per unit of time is

$$TEC=\frac{K+cQ+hQ^2/2a}{Q/a}=aK/Q+ac+hQ/2$$

A solution can be derived analytically by the use of elementary differential calculus. (see Appendix 1). The optimum value of Q, denoted Q_0 which minimize TEC is found to be

$$Q_0=\sqrt{\frac{2aK}{h}} \tag{1}$$

It is very clear that this formula doesn't apply to all economic conditions, it is only the optimum value which combines the setup cost and the holding cost. When we are going to determine the order quantity practically, we have to consider about the restrictions involved. It will be discussed later. The simple formula provides the base in addition to which we may add other conditions to find out the practical order quantity.

Similarly, the corresponding optimum value of the time that take to diminish this optimum value of Q_0, say t_0 is given by

$$t_0=Q_0/a=\sqrt{\frac{2K}{ah}} \tag{2}$$

2. 2. *Ordering or Manufacturing Cost.*

The cost of ordering or manufacturing a run size, say an amount Q can be expressed as K+cQ Where c represents the unit price paid including raw-material cost plus processing cost per unit which is directly proportional to the amount ordered. The other symbol K is often referred to as the setup cost and is constant per production run but is a variable cost per unit. It generally includes shop setup costs, office setup costs; administrative cost of manufacturing ordering, all other production planning and control costs related to the clerical operations of production orders, scheduling, dispatching, instruction, and progress inspection, and the preliminary labor and other expenses of starting a production run. K also including the takedown cost, and involves the cost of entering the finished units in stock and performing the necessary paper work attached thereto, this also may include the cost of the analysis performed by the cost analysis section.

Here variable K is very important, since the economic lot size is proportional to its square root. Many books didn't say clearly what this figure really was and how to get this figure. All what they said was to suppose that the ordering cost is say $350 or the setup cost per run is $350.

It was stated in the last paragraph that this variable K is not only the setup cost, of course it is one of the major components, but also includes the take down cost and all other production and control costs.

This K is referred to as manufacturing cost per run. Usually they divided all cost related to the manufacturing function by the number of production orders issued $K=y/n$, where y is the total ordering cost and n is the number of the lot size.

A mathematical proof will show that this is incorrect, (see Appendix 2).

The total manufacturing cost can be thought of as if it is made up of two parts: one part F is not affected by the number of lot size n; this part can be described as the "fixed" element in manufacturing cost. The other part varies directly with the lot size n; this part can be described as the "variables" element in manufacturing cost.

Then we get equation $y=F+Kn$

If we use the data actually gotten and plot it, we will get a graph as follows.

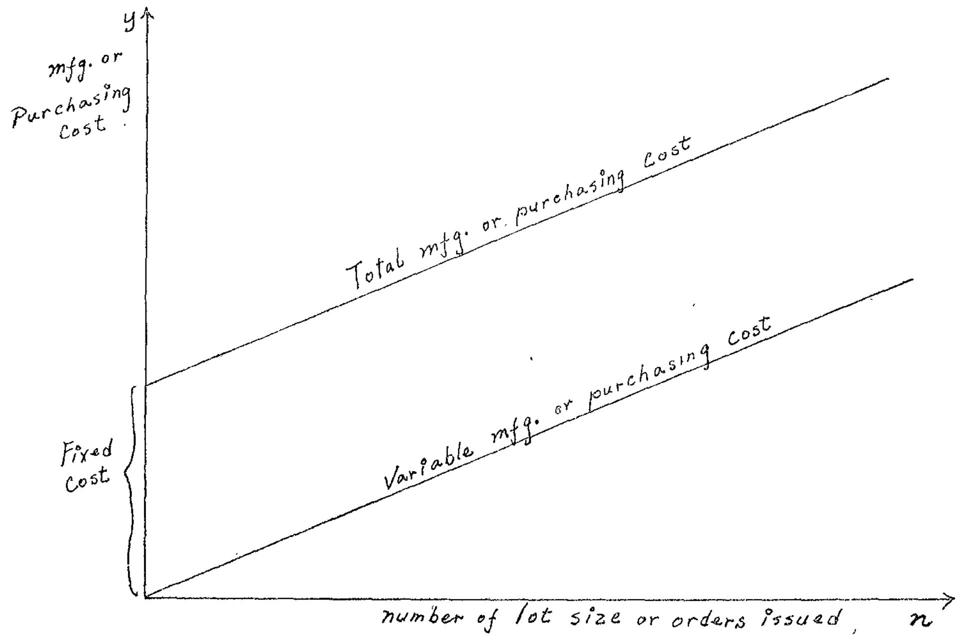

Fig. 2

Fig. 2 is a graph showing the relation-ship between those four variables. The vertical scale on th is graph shows the total manufacturing cost: the horizontal scale shows the number of the lot size. The equation is a linear equation, in practice, however, it is not a linear. That means, the total cost of manufacturing is not quite linear to the number of the lot size. It is only an approximation but the assumption of a linear relationship is convenient for use in estimating the figure K.

From the equation we know K is the slope of the line represented on Fig. 2.

The slope is very easy to find out by an algebraic equation since it is the straight-line relationship between manufacturing cost and number of lot.

$$\text{Slope}=\frac{y_2-y_1}{n_2-n_1}=K$$

The other method to get K is by using the so-called method of least squares, expressed as follows:

$$K=\frac{\sum_{i=1}^{n}(y-\bar{y})(n-\bar{n})}{\sum_{i=1}^{n}(n-\bar{n})^2}$$

Where $i=1,2,3,\cdots\cdots\cdots\cdots,n$

y=the total ordering or manufacturing cost

$\bar{y}$=the arithmetic average of y

n=number of the production lot or purchasing order

$\bar{n}$=the arithmetic average of n

2. 3. *The Holding or Storage Costs.*

This is the value of the annual cost of having an inventory and is expressed as a decimal fraction. It refers to the value of the inventory. This figure represents the costs associated with the storage of the inventory until they are sold or used. They are made up of several items. The most important is the cost of the capital tied up in inventory. Another important item is the cost of storage. By cost of storage we mean all of the expenses incurred by having material in a storeroom, such as the cost of space, salaries of the people responsible for the storeroom, supervisors, material handling crews, clerks, typists, etc. Other important expenses of storage are insurance against fire and theft, protection, and taxes attributed to the materials stored. The annual cost of storing can be obtained from the cost accounting records. The cost of the capital tied up in inventory is usually taken as the cost of money in the market in which the company operates.

The money tied up in inventory, like all of the money used in the company, comes from different sources, It seldom happens that the money comes from a single source. It comes from vendors' credit, short or long term loans, and the owner's equity. If we want to use the right figure we have to take those origins into account.

Suppose the capital invested in the company has four origins, the vendor's credit accounting for 10% of the total capital, short-term loans 20%, long-term loan 30%, and net worth 40%. Let's assume the rate of interest attached to each one of those sources of capital. Vendor's supplies: 1% a month or 12% a year. Short-term loan 18% a year. Long-term loan 6% a year. Net worth 10% (means 10% profitabily a year).

We multiply the rate of interest by the percentile shares of capital:

$$12\%\times 10/100=1.2$$

$18\% \times 20/100 = 3.6$

$6\% \times 30/100 = 1.8$

$10\% \times 40/100 = \underline{4.0}$

Total 10.6

10.6% is the annual cost of capital tied up in inventory for this company. The total cost of getting and keeping an inventory is annual cost of storage plus annual cost of capital tied up in inventory. In this case it will be 20.6% if the cost of storage is 10%.

3. 1. *Economic Lot Size with Shortage.*

If shortage is allowed, and let u be the shortage cost for each unit for a unit of time. This inventory situation can be summarized in Fig 3, where S is the inventory level at the beginning of each period.

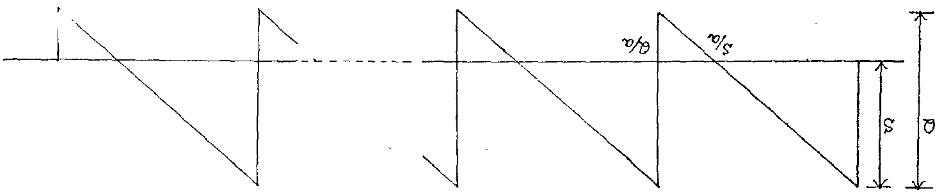

Fig. 3

The production cost per run is given by

$$\begin{array}{ll} O & \text{if } Q=0 \\ K+cQ & \text{if } Q>0 \end{array}$$

The holding cost per run is given by

$$h\int_0^{S/a} S(t)dt = h\int_0^{S/a}(S-at)dt = h\Big[St - 1/2\ at^2\Big]_0^{S/a}$$

$$= h[S(S/a) - 1/2\ a(S^2/a^2)] = hS^2/2a$$

Simiarly the shortage cost per run is given by

$$u\int_0^{Q/a-S/a} atdt = u\int_0^{Q/a-S/a} 1/2\ at^2 = u\Big[1/2\ at^2\Big]_0^{Q/a-S/a} = u[1/\quad a(Q/a-S/a)^2] = u(Q-S)^2/2a$$

Therefore the total cost per run is

$$K + cQ + hS^2/2a + u(Q-S)^2/2a$$

and the total cost per unit of time is

$$TEC = \frac{K + cQ + hS^2/2a + u(Q-S)^2/2a}{Q/a}$$

$$= aK/Q + ac + hS^2/2Q + u(Q-S)^2/2Q$$

The optimum value of Q and S can be derived (see Appendix 3). The result we get are

$$Q_0 = \sqrt{2aK/h}\sqrt{(u+h)/u} \qquad (3)$$

$$S_0 = \sqrt{2aK/h}\sqrt{u/(u+h)}$$

The optimum interval between production run t_0 is given by

$$t_0 = Q_0/a = \sqrt{2K/ah}\sqrt{(u+h)/u} \qquad (4)$$

3. 2. *Unsatisfied Demand or Shortage Penalty Cost.*

This cost is created by the discrepancy between demand and supply. It is incurred when the stock of the commodity proved to be inadequate to meet the demand, through either a delay in meeting demand or the inability to meet it at all. In the first case demand may be met by a priority shipment. In this case the penalty cost can be viewed as the difference between the cost of priority shipment and the cost of routine delivery. The second case is not meeting the demand, but meeting it when the supply is available again. The cost of a shortage penalty in this case will be the loss of customer's goodwill. The loss of customer's goodwill cannot be measured in terms of dollars. It is very difficult to assign the cost, but we can use a monetary scale in all research by weighting objectives of so-called intangibles. This cost involves all profit-loss that we can avoid, if we have enough inventory.

3. 3. *The Salvage Cost*

The salvage cost is the opposite of the salvage value that is the value of a left over item at the end of inventory period. If the period continue for ever and no obsolescence happens then the item is still available for the next period. When the disposal cost of an item exceeds its value the difference is the salvage cost.

3. 4. *The Discount Rate*

The discount rate takes into account the time value of money. When a company ties up capital in inventory, it is prevented from using this money for alternative purposes. It could invest this money in secure investments, say government bonds, and have a return on investment, a year hence, of say 5%. Thus a dollar invested today would be worth $1.05 a year hence. Therefore a dollar profit a year hence is equivalent to $x=\frac{1}{1.05}$ dollars today. This figure x is known as the discount factor and should multiply it to profit one year hence, when you consider "effectiveness". We may also take into account the opportunity cost in our consideration. If the money tied up in inventory is available to invest on an alternative and the rate of return will be say, 20% of the capital invested, we may determine this figure as the opportunity cost.

4. *Economic Lot Size with Production Time.*

So far we have considered instantaneous production. Now, practically speaking, there is also production time and the inventory will accumulate day by day when the production is going on. In this case if no shortage is allowed, the inventory situation can be summarize in Fig. 4 Now let us consider that there is some production time-say Q/b where b is the number of units produced in each unit time. The production continues to the time Q/b and then stops, when the inventory is at its maximum of $S=Q-\frac{a}{b}Q$, after that the inventory will decrease at a rate a until time Q/a. The inventory situation can be summarized in Fig-4. The production cost per

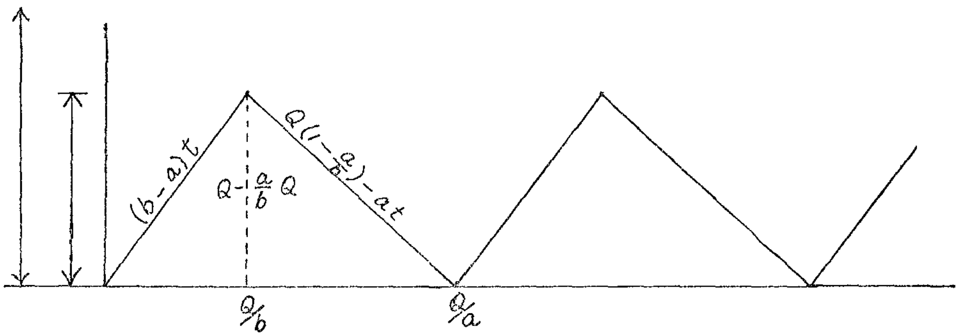

Fig. 4

run is given by

$$\begin{cases} 0 & \text{if } Q=0 \\ K+CQ & \text{if } Q>0 \end{cases}$$

The holding cost per run is given by

$$h/2(1-a/b)Q^2/a=hQ^2/2a\ (1-a/b)$$

Therefore the total cost per run is

$$K+cQ+hQ^2/2a\ (1-a/b)$$

And the total expected cost per unit of time is

$$TEC=\frac{K+cQ+hQ^2/2a-hQ^2/2b}{Q/a}=aK/Q+ac+hQ/2\ (1-a/b)$$

The value of Q_0 which minimize the TEC is obtained by analytic solution given in Appendix 4.

$$Q_0=\sqrt{\frac{2aK}{h(1-a/b)}} \tag{5}$$

The corresponding optimum interval between production run is

$$t_0=\sqrt{\frac{2K}{ah(1-a/b)}} \tag{6}$$

5. *Inventory Model with Restriction.*

Up to now we discussed some economic lot size equations that are without restrictions, but in the real situation we have to consider some restrictions on capital, machine hours, storage places, equipment and personnel etc.

We may introduce these restrictions to the situation that involve more than one product. In intermittent manufacturing a wide variety of products are produced to order. In continouus manufacturing there are also joint products. Here we want to discuss the model that enables us to determine how much of each product should be produced under the specified restrictions. It is our concern to get high capita productivity of labor through maximizing profits or minimizing coste. This in turn depends on the effective use of assets. The proportion of fixed assets to total assets will vary from industry to industry and from enterprise to enterprise within an industry. Fixed assets may include Land, Buildings and Structures, Machinery Equipment and Tools, Transportation systems etc. depending on the kind of industry. It is not realistic to discuss a model developed for controlling inventory in situations where no restrictions exist. Any concern has limited resources of assets which will effect the course of action in determining the economic lot size that minimize the total production cost.

5. 1. *Economic Lot Size: No Restriction*

Consider a manufacturer who produces discrete products X_i (i=0, 1, 2,......n) and supplies R_i units to his customers during a month (the demand is known and constant). Then the lots number will be $\frac{R_i}{Q_i}$ per month on the average.

The production cost per run is given by

$$0 \quad \text{if } Q_i=0$$
$$K_i+C_iQ_i \text{ if } Q_i>0$$

The unit cost associated with the setup is K_i/Q_i plus the prime cost of the item C_i will be the value per unit of inventory. The average level of inventory will be $Q_i/2$. Therefore the holding cost per run is given by

$$Q_i/2\ (K_i/Q_i+C_i)\ h_i=h_iK_i/2+h_iC_iQ_i/2$$

Consequently, the total expected cost per month of time is

$$TEC=R_i/Q_i\ (K_i+C_iQ_i)+h_iK_i/2+h_iC_iQ_i/2 \qquad (7)$$

The value of Q_{i0} which minimize the TEC is obtained by the analytic solution given in Appendix 5.

$$Q_{i0}=\sqrt{2R_iK_i/h_iC_i} \qquad (8)$$

5. 2. *Economic Lot Size with Space Restriction.*

The last section represents the optimum lot which will yield the minimum cost without restriction. Here we consider a concern which has limited warehouse space. The storage of inventory will not be allowed to exceed this available space, say S. If one unit of product i requires W_i cubic foot, the total space required will be the total number of products multiplied by this figure W_i.

Churchman stated that the average total space required will be $1/2\ \sum_i W_iQ_i$, since the average space occupied by product i is $1/2\ Q_i$. He used this figure $1/2\ \sum_i W_iQ_i$ as the space required*.

In practice it is incorrect. It is impossible to have this inventory $1/2\ Q_i$ all the time during the period. In the models stated previously we consider two kinds of production. The first production is instantaneous, that is represented graphically as is done in both Fig. 1 and Fig. 3. The second production is not instantaneous one; there is some length of production time. See Fig. 4. Both instantaneous and noninstantaneous production have their maximum inventory. In instantaneous case the maximum inventory will be Q_i since there is no duration of production time,

*Churchman' Ackoff and Arnoff, "Introduction to Operations Research" p.261

while in the noninstantaneous case the maximum inventory during the period is $Q_i - a/b\ Q_i$ where the production period is less than the period between each production lot, i.e. $Q_i/b < Q_i/a$. The shorter the production time the closer the maximum inventory will be to Q_i. Practically it is very rare to use 100% of the available space, thus the total space required should be more than $\sum_i W_iQ_i$ or at most equal to it.

Here if S is the total available space for storage, we require that

$$\sum_i W_iQ_i \leqq S$$

Then

$$S - \sum_i W_iQ_i > 0$$

Or

$$S - \sum_i W_iQ_i = 0$$

Define the quantity λ such that

$$\lambda = 0 \text{ when } S - \sum_i W_iQ_i > 0$$

$$\lambda < 0 \text{ when } S - \sum_i W_iQ_i = 0$$

Thus $\lambda(S - \sum_i W_iQ_i)$ is always zero and may be added to the cost equation without changing the value of TEC.

$$TEC = \sum_i R_i/Q_i\ (K_i + C_iQ_i) + \sum_i h_iK_i/2 + \sum_i h_iC_iQ_i/2 + \lambda(S - \sum_i W_iQ_i) \quad (9)$$

TEC is minimized by finding partial derivatives with respect to the Q_i and equating the derivatives to zero.

$$\partial TEC/\partial Q_i = -R_iK_i/Q^2_i + h_iC_i/2 - \lambda W_i$$

Therefore

$$Q_{i0} = \sqrt{2R_iK_i/(h_iC_i - 2\lambda W_i)} \quad (10)$$

For each product R_i, K_i, h_i, C_i andW_i are known and it is necessary to find the value of λ.

Assuing that space may be rented at the price of \$E per cubic foot per month, we may get the new cost equation which includes rental for Storage space.

$$TEC = \sum_i R_i/Q_i\ (k_i + C_iQ_i) + \sum_i h_iK_i/2 + \sum_i h_iC_iQ_i/2 + E\sum_i W_iQ_i \quad (11)$$

The partial derivative of TEC with respect to Q_i has changed to

$$\partial TEC/\partial Q_i = -R_iK_i/Q_i + h_iC_i/2 + EW_i$$

Setting the derivative equal to zero and solving for optimum Q_i, we obtain

$$Q^*_{i0} = \sqrt{2R_iK_i/(h_iC_i + 2EW_i)} \quad (12)$$

5. 3. *Space cost*

In the last section we have another cost called space cost which is a function of economic lot size subject to storage space restriction. This cost, denoted as *E* in dollars per cubic foot, is not included in the holding cost h in this case. As stated in the section 2. 2. We define holding cost as the cost of having an inventory and the cost of storage. They include personal property taxes, insurance, obsolescence, deterioration from handling, interest on money invested, maintenance, salaries of the people responsible for the store room, and anything else which is proportional to the value of inventory. *E* is space cost per unit of space including rent, heat, light, etc. Comparing the equations (8) and (12), we see that $\overset{*}{Q}_{i0}$, the economic lot size with space restriction, is smaller than Q_{i0}, the economic lot size with no restriction.

$$\sqrt{\frac{2R_iK_i}{h_iC_i+2EW_i}}<\sqrt{\frac{2R_iK_i}{h_iC_i}} \qquad \text{Since } EW_i>0$$

Consequently the total expected cost of the economic lot size with space restriction is greater than that with no restriction. We will find out that it will profit the company to rent additional warehouse space for that value of E which satisfies the following conditions,

$$E\Sigma W_iQ_i=\lambda(S-\Sigma W_iQ_i) \qquad (13)$$

As stated before the right side of the equation is always zero where $\lambda<0$, and $S-\Sigma W_iQ_i=0$, or $\lambda=0$, when $S-\Sigma W_iQ_i>0$

Here we don't consider the case $\lambda=0$, when $S-\Sigma W_iQ_i>0$ since it is a case with no space restriction.

Rewrite the equation (13) as

$$E\Sigma W_iQ_i+\lambda\Sigma W_iQ_i=\lambda S$$

$$(E+\lambda)\Sigma W_iQ_i<0$$

Since ΣW_iQ_i is always positive,

Then $E+\lambda<0$

$E<-\lambda$

Thus if E is lesser than $-\lambda$, it will profit the company to rent additional warehouse space in order to operate at Q_{i0} that minimizes the total expected cost.

From equation (10), for any arbitrarily assigned value of λ, Q_i and, hence, ΣW_iQ_i can be calculated. The value of λ can be calculated by the trial and error method where calculated ΣW_iQ_i is equal to S, the available space.

APPENDIX 1

In this appendix, we determine that value of Q, denoted by Q_0, which minimize the total expected cost TEC(Q), where

$$TEC(Q)=aK/Q+ac+hQ/2$$

Proceeding by the use of calculus,

$$dTEC/dQ=-aK/Q^2+h/2$$

So that, setting this derivative equal to zero, we get

$$aK/Q^2_0=h/2$$

$$Q^2_0=2aK/h$$

$$Q_0=\sqrt{2aK/h}$$

Therefore

$$t_0=Q_0/a=\sqrt{2aK/h}\sqrt{1/a^2}=\sqrt{2K/ah}$$

APPENDIX 2

The economic lot size we derived from Appendix 1 is

$$Q_0=\sqrt{2aK/h}$$

Where　K=ordering cost or manufacturing cost

h=holding cost

a=withdrawn rate

Assume R=Demand during time T

Then $a=R/T$

Therefore $Q_0=\sqrt{2RK/hT}$

Suppose y=Total annual manufacturing or purchasing cost

n=number of orders issued per year

Then $n=R/Q_0$

If $K=y/n=yQ_0/R$

Therefore $KR=yQ_0$

Substitute to the upper economic lot size equation we will get

$$Q_0=\sqrt{2yQ_0/hT}$$

This is meaningless.

APPENDIX 3

In this appendix, we determine the values of Q and S, denoted by Q_0 and S_0; which minimize the total expected cost TEC where

$$TEC=aK/Q+ac+hS^2/2Q+u(Q-S)^2/2Q$$

Proceeding by the use of calculus, we get

$$\partial TEC/\partial S=hS/Q-u+uS/Q$$

$$\partial TEC/\partial Q=-aK/Q^2-hS^2/2Q^2+u/2-uS^2/2Q^2$$

Setting these partial derivatives equal to zero and simplifying, we obtain

$$S_0=uQ_0/\ (u+h)$$

$$Q_0^2=\frac{2aK+(u+h)S_0^2}{u}$$

Solving this system of equations for S and Q, we obtain

$$Q_0^2=2aK(u+h)/hu$$

Therefore $Q_0=\sqrt{2aK/hu}\sqrt{(+h)/u}$

Hence $S_0=uQ_0/(u+h)$

$$S_0=\sqrt{2aK/h}\sqrt{u/(u+h)}$$

APPENDIX 4

In this appendix, we determine that value of Q, denoted by Q_0, which minimize the tota expected cost TEC(Q) where

$$TEC(Q)=aK/Q+ac+hQ/2\ (1-a/b)$$

Proceeding by the use of calculus

$$dTEC/dQ=-aK/Q^2+h/2\ (1-a/b)$$

So that, setting derivative equal to zero, we get

$$aK/Q_0^2=h/2\ (1-a/b)$$

$$Q_0^2h\ (1-a/b)=2aK$$

$$Q_0^2=2aK/h(1-a/b)$$

$$Q_0=\sqrt{2aK/h(1-a/b)}$$

Therefore $t_0=Q_0/a=\sqrt{2K/ah(1-a/b)}$

APPENDIX 5

In this appendix, we determine that value of Q_i, denoted by Q_{i0}, which minimize the total expected cost $TEC(Q_i)$ where

$$TEC(Q_i)=R_i/Q_i\ (K_i+C_iQ_i)+h_iK_i/2+h_iC_iQ_i/2$$

Proceeding by the use of calculus

$$dTEC/dQ=-R_iK_i/Q_i^2+h_iC_i/2$$

So that, setting the derivative equal to zero, we get

$$R_iK_i/Q^2_{i0}=h_iC_i/2$$

$$Q^2_{i0}=2R_iK_i/h_iC_i$$

$$Q_{i0}=\sqrt{2R_iK_i/h_iC_i}$$

BIBLIOGRAPHY

1. Churchman, C. W., Ackoff, R. L., and Arnoff E.L., *Introduction to Operations Research*, John Wiley & Sons, Inc. 1957
2. Magee, John F., *Production Planning and Inventory Control*, McGraw-Hill Book Company, Inc. 1958
3. Sasieni, M.W., Yaspan, A., and Friedman, L., *Operations Research: Methods and Problems*, John Wiley & Sons, Inc. 1959
4. Ireson, W.G., and Grant, E. L., Handbook of Industrial Engineering *and Management*, Prentice Hall, Inc. 1955
5. Yokoyama, and Hukuba, *Inventory Control*, Kyorichu. 1965

Inventory Models and Cost Variables

Albert T.Y. Kuo

Even though many models for the determination of eonomic lot size have been demonstrated, yet few companies are willing to use them to use them to balance the inventory holding cost and setup cost quantitatively.

One of the reasons is that it is very difficult to determine the necessary cost variables correctly. Furthermore, their definitions are also indefinite.

Because it is really very difficult to grasp the correct cost variables in practice, therefore it seems very important to refer to the concept of strictly analysed economic lot size.

The objective of this paper is therefore to define those cost variables by employing the use of some models of economic lot size for manufacturing and purchasing.

存量模式及成本變數

郭 東 耀

雖然許多決定經濟批量的模式業已證明成立，却僅少數的公司願意使用它們以數量上達到存貨儲備費用與製造辦事費用的均衡。其原因之一是很難正確的決定必要的成本變數。此外它們的定義也混淆不清。由於從實際上要瞭解正確的成本變數很難，所以對嚴格分析的經濟批量概念之介紹是很重要的。此篇論文的目的即在應用一些經濟批量的模式，以對那些有關製造和採購的成本變數作一明確的定義。